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Cabibbo Angle II

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When permutation symmetry and phase invariance are assumed, the mass matrices for two generations take a unique form. The expressions for the masses of various quarks found earlier are used in the mass matrices to determine the Cabibbo angle. This angle is found to be 13° 11′. It is very close to the experimental value.

1. INTRODUCTION

Recently I derived expressions for the masses of the charged leptons e and μ (Raju, 1987a):

$$m_e^2 = m_1 M_{\mathrm{WL}} \frac{(g_v/g_A)^4_{\nu_e}}{(g_v/g_A)^4_{e\mu}} \left\{ 1 - \left[1 - \left(\frac{g_v}{g_A} \right)^4_{e\mu} \right]^{1/2} \right\} \tag{1}$$

$$m_\mu^2 = m_1 M_{\mathrm{WL}} \frac{(g_v/g_A)^4_{\nu_e}}{(g_v/g_A)^4_{e\mu}} \left\{ 1 + \left[1 - \left(\frac{g_v}{g_A} \right)^4_{e\mu} \right]^{1/2} \right\} \tag{2}$$

where m_1 is the rest mass of the electron or muon neutrino. I arrived at equations (1) and (2) by arranging $m_{\nu_e} = m_{\nu_\mu} = m_1$. The mass of the neutrino is given by

$$m_1 = \frac{m_e m_\mu}{M_{\mathrm{WL}}} \left(\frac{g_v}{g_A} \right)^2_{e\mu} \simeq 6.5 \text{ eV} \tag{3}$$

If $M_{\mathrm{WL}} = 80$ GeV. In these equations g_v and g_A denote the vector and axial vector couplings of the particles denoted by the subscripts to the Z-boson of the standard model. From equations (1) and (2) we also note that,

$$\frac{2 m_e m_\mu}{m_e^2 + m_\mu^2} = \left(\frac{g_v}{g_A} \right)^2_{e\mu} \tag{4}$$

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and with the precisely known values of m_e (0.510 MeV) and m_μ (105.7 MeV) we find that

$$\sin^2\theta_W = 0.2254 \text{ or } 0.2746 \tag{5}$$

The first value agrees very well with the world average of $x_W = \sin^2\theta_W$ determined from various experiments of electroweak physics. The second value was interpreted elsewhere (Raju, 1986) as the mixing parameter $x_R = e^2/g_R^2$ which occurs in a gauge model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$, where g_R is the gauge constant of $SU(2)_R$. Here e is the electromagnetic coupling constant.

In analogy to equations (1) and (2), I also found expressions for the constitutent masses of all the quarks (Raju, 1986):

$$m_c^2 = m_d M_{\mathrm{WL}} \frac{(g_v/g_A)^4_{dsb}}{(g_v/g_A)^4_{uct}} \left\{ 1 - \left[1 - \left(\frac{g_v}{g_A} \right)^4_{uct} \right]^{1/2} \right\} \tag{6}$$

$$m_t^2 = m_d M_{\mathrm{WL}} \frac{(g_v/g_A)^4_{dsb}}{(g_v/g_A)^4_{uct}} \left\{ 1 + \left[1 - \left(\frac{g_v}{g_A} \right)^4_{uct} \right]^{1/2} \right\} \tag{7}$$

The similarity between expressions (1) and (2) on one hand and the expressions (6) and (7) on the other is striking. In all these equations g_v/g_A is the ratio of the vector to axial vector couplings of the particles indicated by the subscripts to the neutral Z-boson of the standard model. We also found that

$$m_s^2 = m_u M_{\mathrm{WL}} \frac{(g_v/g_A)^4_{uct}}{(g_v/g_A)^4_{dsb}} \left\{ 1 - \left[1 - \left(\frac{g_v}{g_A} \right)^4_{dsb} \right]^{1/2} \right\} \tag{8}$$

$$m_b^2 = m_u M_{\mathrm{WL}} \frac{(g_v/g_A)^4_{uct}}{(g_v/g_A)^4_{dsb}} \left\{ 1 + \left[1 - \left(\frac{g_v}{g_A} \right)^4_{dsb} \right]^{1/2} \right\} \tag{9}$$

In equations (6)-(9), m_d and m_u are the constituent masses of down and up quarks, respectively. For numerical calculations we assume that $m_u \simeq m_d = 0.3$ GeV. From standard model prescription we know that

$$\begin{aligned} (g_v/g_A)_d^2 = (g_v/g_A)_s^2 = (g_v/g_A)_b^2 = (-1 + \tfrac{4}{3}x_L)^2 \\ (g_v/g_A)_u^2 = (g_v/g_A)_c^2 = (g_v/g_A)_t^2 = (-1 + \tfrac{8}{3}x_L)^2 \end{aligned} \tag{10}$$

Here $x_L = \sin^2\theta_W$, where θ_W is the Weinberg mixing angle. From equations (6)-(9) we obtain $m_c = 1.7$ GeV, $m_t = 21.231$ GeV, $m_s = 0.57$ GeV, and $m_b = 2.18$ GeV, provided $M_{\mathrm{WL}} = 80$ GeV. With these values I found an expression for the Cabibbo angle (Raju, 1987b). It turned out to be $13°42'$, which is an excellent result. In the present article I wish to examine this calculation of the Cabibbo angle from the point of view of permutation invariance and phase invariance. When these invariances are assumed the value of the

Cabibbo angle turns out to 13° 11′, which is much closer to the experimental value than that reported in Raju (1987b) henceforth referred to as I).

2. THE MASS MATRICES

Let the total number of Higgs multiplets that couple to quarks be at most two, so that the electroweak groups is $SU(2)_L \times SU(2)_R \times U(1)$. If there are two generations of fermions, the full gauge-invariant Lagrangian is invariant under permutations among the different generations, provided the Higgs sector is excluded. We can also allow a phase transformation among the various generations of fermions, such that

$$f_i = e^{i\alpha_i} f_i \tag{11}$$

where i is the generation number and α_i is an arbitrary phase factor. In particular, if we confine our attention only to quarks, equation (11) shows that each generation of quarks transforms into itself, times a phase factor. This is very close to the situation that I considered for the charged leptons e and μ in Raju (1987a). All the Higgs fields must have well-defined properties under the phase symmetry,

$$\phi_i = e^{i\chi_i} \phi_i \tag{12}$$

where χ_i is also arbitrary. Yukawa couplings are allowed if they are invariant under the phase symmetry. We now try to construct the mass matrices for quarks on the above basis of permutation symmetry and phase transformation. For simplicity we assume only two generations of quarks.

The above requirements lead surprisingly to a unique nontrivial form of the mass matrix for two generations. For u, c quarks we have

$$M_1 = \begin{pmatrix} 0 & b_1 \\ b_1 & c_1 \end{pmatrix} \tag{13}$$

The entries are assumed here real for simplicity, although this is not necessary in general. The mass squared matrix $M_1^2 = M_1^\dagger M_1$ is diagonalized by an orthogonal matrix $R_1(\theta_1)$, where

$$\tan 2\theta_1 = 2b_1/c_1 \tag{14}$$

The entries of (13) are given by

$$b_1^2 = m_u m_c = 0.51 \text{ GeV}^2 \tag{15}$$

$$c_1^2 = (m_c - m_u)^2 = 1.96 \text{ GeV}^2 \tag{16}$$

The eigenvalues of M_1^2 are m_u^2 and m_c^2, where the latter is given by equation (6). In computing (15) and (16) we took $m_u = 0.3$ GeV and $m_c = 1.7$ GeV obtained for m_c from equation (6). From equations (14)-(16) it is clear that

$$\theta_1 = \tan^{-1}(m_u/m_c)^{1/2} = 22° \, 47' \tag{17}$$

A similar analysis for d, s quarks shows that

$$M_2 = \begin{pmatrix} 0 & b_2 \\ b_2 & c_2 \end{pmatrix} \tag{18}$$

The eigenvalues of $M_2^2 = M_2^\dagger M_2$ are given by m_d^2 and m_s^2, where m_s^2 is given by equation (8), provided

$$b_2^2 = m_d m_s = 0.171 \text{ GeV}^2 \tag{19}$$

$$c_2^2 = (m_s - m_d)^2 = 0.0729 \text{ GeV}^2 \tag{20}$$

This M_2^2 matrix is diagonalized by the orthogonal matrix $R_2(\theta_2)$, where

$$\tan 2\theta_2 = 2b_2/c_2 \tag{21}$$

From equations (19)-(21) we readily observe that

$$\theta_2 = \tan^{-1}(m_d/m_s)^{1/2} = 35° \, 58' \tag{22}$$

where we took $m_d = 0.3$ GeV and the value of m_s calculated from (8) is used.

3. THE CABIBBO ANGLE

As shown in 1, the Cabibbo angle is obtained from the orthogonal matrix

$$R(\theta_C) = R_2^{-1}(\theta_2) R_1(\theta_1) \tag{23}$$

From equations (17) and (22) we find that

$$\theta_C = \theta_2 - \theta_1 = 13° \, 11' \tag{24}$$

and

$$\sin \theta_C = 0.228 \tag{25}$$

4. CONCLUSIONS

The result given here agrees pretty well with experiment. Compared to the value of the Cabibbo angle evaluated in 1, the present value is identical to the experimental value. Apart from this, this result is obtained on the basis of permutation and phase invariance among fermion generations. In this respect the present note is rich is physics compared to 1.

The extension of these results to three generations is straightforward, but the matrices so obtained consist of two categories. The mass matrices are not unique. To make these mass matrices unique with three generations we may have to postulate some more assumptions. Moreover, the third generation quark masses m_t and m_b evaluated here through equations (7) and (9) appear to be inconsistent with experiment (Schwarzschild, 1987). However, we will examine the three-generation case separately.

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